Retardation of Relaxation Solution Methods

EFFECTS OF LIMITING CONDUCTANCE

Eigenvalue Spectrum of A

- While relaxation appears to offer an efficient rapid solution method, the test case was selected to optimize convergence rate in SOR:
  - ➢ Baseline: all CV were designed with the same conductivity
    - aluminum
    - 200 W/m C
  - ➢ cold sink condition connected with same conductance as all other CV

➥ ***An iterative solution proceeds in much the same fashion as the physical process because of the similarities between the diffusion of numerical quantities and diffusion of physical energy.***

EFFECTS OF LIMITING CONDUCTANCE

Eigenvalue Spectrum of A

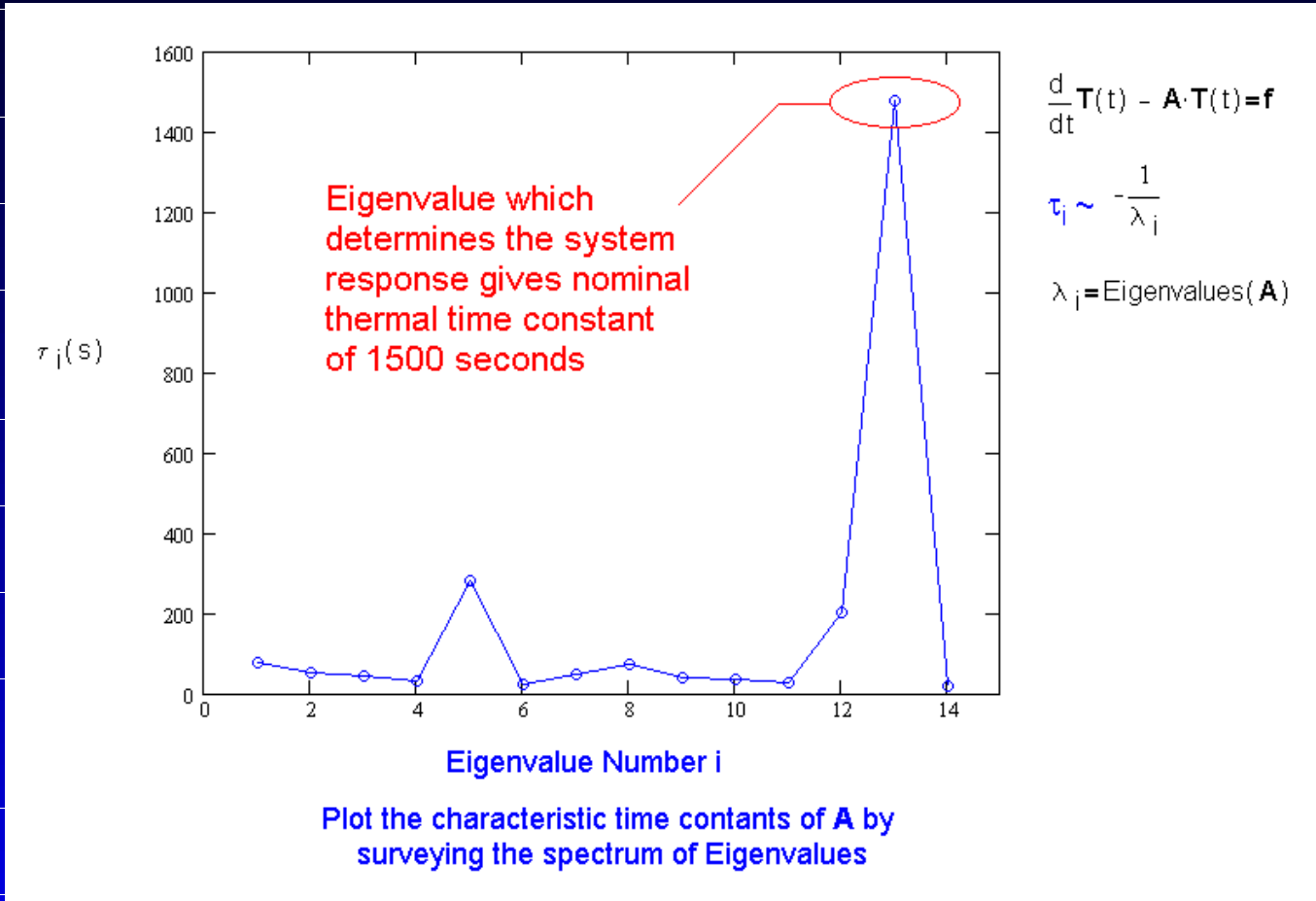
- **Solution Progress Rate**: *related to the Eigenvalues of the matrix **A***
  - ➢ limiting time constant determines physical transient response but also describes the iterative response
  - ➢ recall analogy between an iterative solution and a transient solution

$$\tau_{Limit} = Max\left(\frac{1}{\lambda_i}\right)$$

$$\lambda \sim \frac{\alpha}{\Delta X^2}$$

$$\tau_{Limit} \sim \frac{\Delta X^2}{\frac{k}{\rho C_p}}$$

- Note that there is a range of Eigenvalues with one dominate EV that defines the speed of response

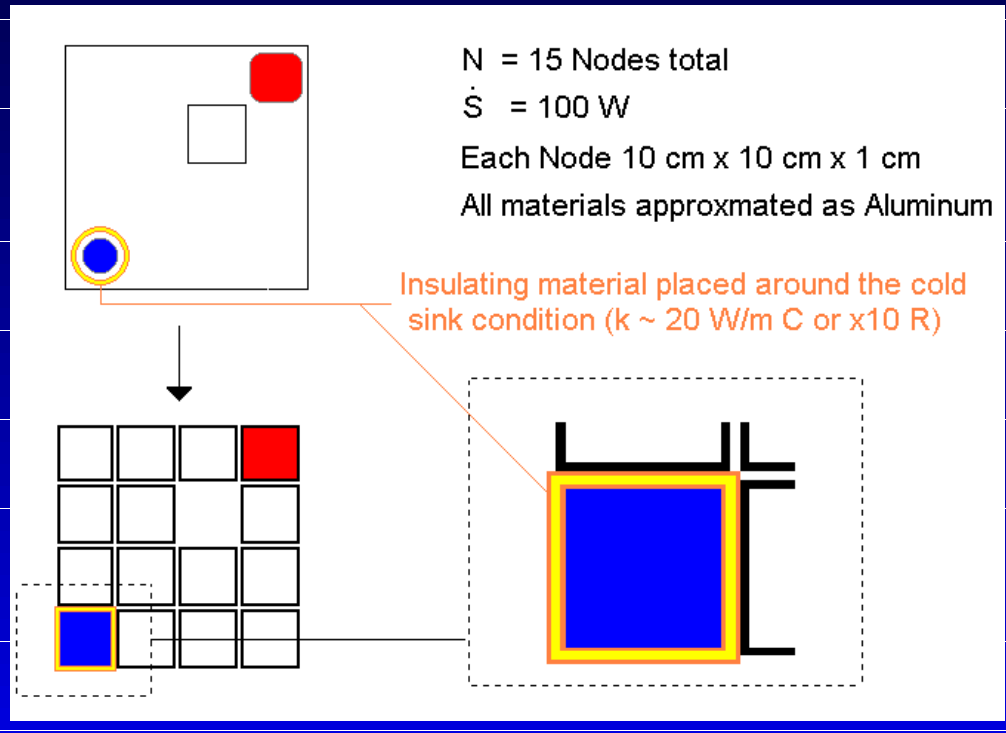


Spectrum of time constants for baseline coldplate example

EFFECTS OF LIMITING CONDUCTANCE
Coldplate Example

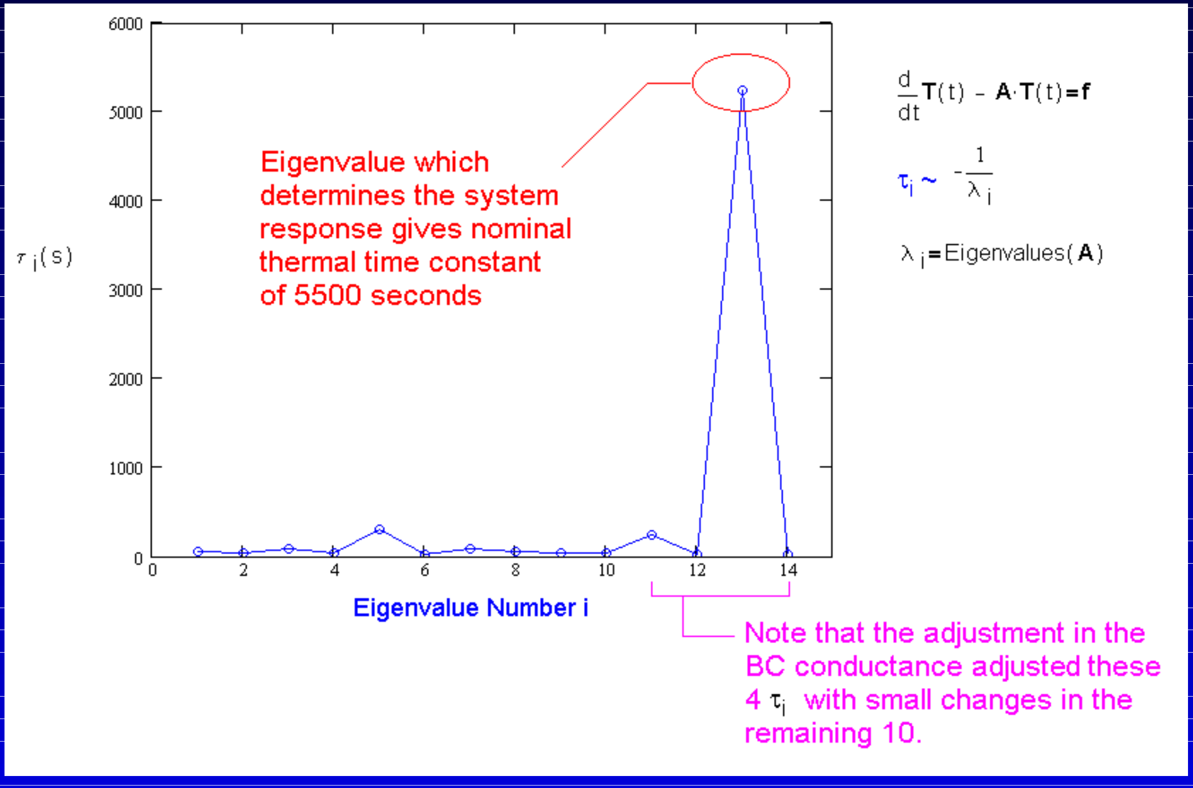
- We can alter the spectrum of Eigenvalues in a CTA discretization by modifying just two conductance values

- Restrict the energy flow to the cold sink by reducing the conductivity around the sink
  - all other CV have same conductance
  - this will only effect two entries in the conductance matrix ***G***
  - demonstrative effect in the spectrum



Revised coldplate example with x10 increase in cold BC thermal resistance

- Simple conductance change creates a marked change in the Eigenvalue spectrum
  - iterative relaxation solutions will then proceed at this lower time constant
  - compare the steady state solution norm as a function of over-relaxation factor



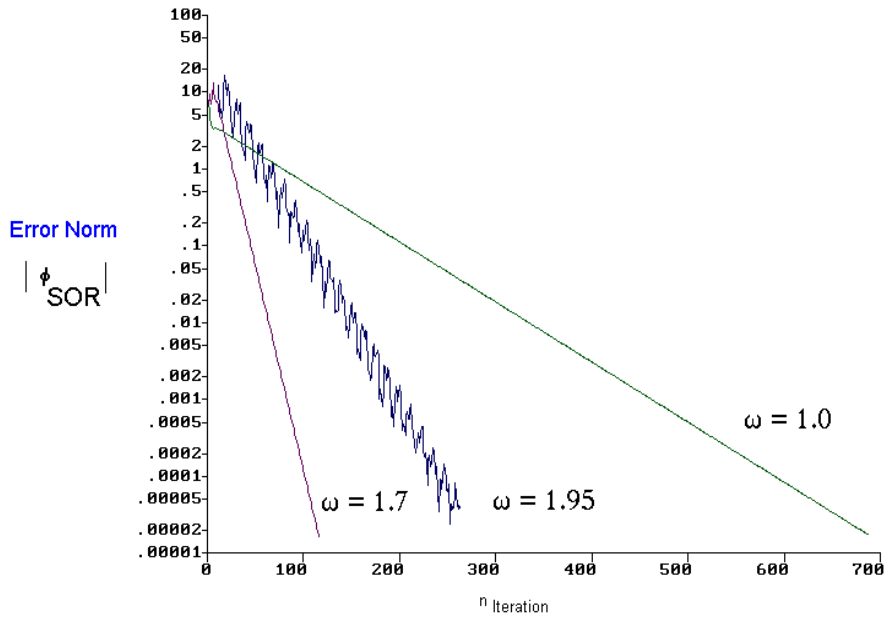
Spectrum of time constants for revised coldplate example with cold BC connected with X10 increase in thermal resistance

EFFECTS OF LIMITING CONDUCTANCE
Coldplate Example

- While the basic form of the residual equations remains unchanged, the SOR solution method requiring additional iterations to achieve the same residual norm
- Iteration response is typical of a thermal simulation where energy is shunted to a boundary condition (sink) with a limiting in-line resistance
- Choking effect: effectively the solution progress of the more conductive aluminum CV are retarded by the resistive features of this single BC

- As demonstrated, relaxation methods can be limited in their application to systems where there is a wide span of conductance values:
  - adjustment of the relaxation factor may have only momentary effects with the solution resuming slow convergence
  - steady state solution times with large 3D models exceeding days are not unreasonable with SOR

Steady state residual error norm for revised coldplate model with x10 increase in BC resistance. Indicates increased iteration count to achieve steady state solutions with speedup with over relaxation.



➥ Under these conditions, the rates of convergence can be arduously slow to the point that relaxation methods becomes unusable

Note the demonstration of the analogy between physical and iterative numerical diffusion process. Iteration (SOR) produces a diffusion-like response. Residuals must diffuse around corners. This can proceed slowly.

ANIMATION DISPLAY

SOR Steady state solution - residual field at each iteration

CONTROL OF SOLUTION PROGRESS
Discussion

➥ ***The failings of SOR have predicated the development of semi-direct solution methods***

- **Semi-Direct**: *methods which offer a quasi-exact inversion of the basic conservation equation*
  - ➢ upon inverting the matrix ***A*** in the basic equation $\frac{d}{dt}T - A\,T = f$ the iterative aspect would be eliminated
  - ➢ each time step would require just one ***approximate*** inversion
  - ➢ by design, this inversion process would be insensitive to the presence of a limiting conductance values