

## About the instructor

Dean Schrage received his B.S. and M.S. degrees in mechanical engineering from *University of Wisconsin - Milwaukee* and his Ph.D. degree in mechanical and aerospace engineering from *Case Western Reserve University*. He is currently working at the *NASA Glenn Research Center*, supporting the *Microgravity Sciences Division*. He has worked almost exclusively in applied research and development in the field of thermal and fluid management - a combined experience totaling 15 years. He is actively immersed in development of CFD and CTA simulation codes and the application of these codes to practical industry problems. He holds 3 U.S. patents and 5 pending acceptance and has authored over 20 journal and conference papers. He has prepared and delivered over 75 technical presentations to customers and has developed and delivered an industrial short course in dynamic simulation methods.

For more information about the CTA course, please visit the web address

[www.TITANAlgorithms.com/courses.html](http://www.TITANAlgorithms.com/courses.html)

or contact the instructor directly at:

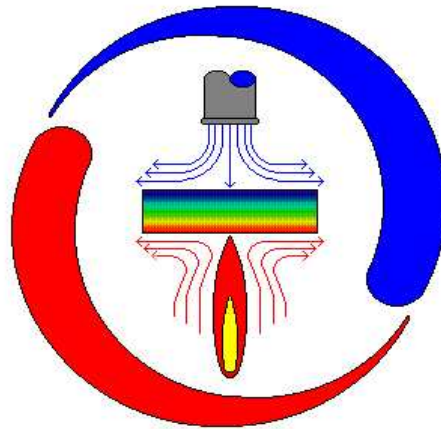
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# CTA

## Computational Thermal Analysis

A Comprehensive Engineering Short Course



Instructor - Dean S. Schrage  
Ph.D. Thermal-Fluid Physics

[www.TITANAlgorithms.com/course.html](http://www.TITANAlgorithms.com/course.html)

## About the Course

*Computational thermal analysis (CTA)* is a critical discipline that drives all stages of the engineering design process. It has evolved as a subdiscipline of engineering analogous to CFD. The present course provides a singular focus on the thermal analysis process, providing a unique perspective by developing all concepts with practical examples. The course will take the student inside the basic simulation codes used today, giving them the ability to understand and appreciate all phases of the thermal modelling process.

## Who Would Benefit

The CTA course is designed as a fundamentals course for the practicing thermal engineer, but is considerably detailed to engage high-level analysts more interested in the theoretical underpinnings of CTA. An introductory knowledge of heat transfer and numerical analysis is required to gain maximum value.

## Course Content

The CTA course is designed around 8 lectures:

- ✓ **Lecture 1** *Introduction to Computational Thermal Analysis (CTA)*
- ✓ **Lecture 2** *Formulation of the Basic Equations of Heat Transfer*
- ✓ **Lecture 3** *Decoupling Systems and Deriving Boundary Conditions*
- ✓ **Lecture 4** *Discretization of the Governing Equations and Geometry*
- ✓ **Lecture 5** *Computational Solutions to the Discrete Equations*
- ✓ **Lecture 6** *Validation of Computational Models and Solutions*
- ✓ **Lecture 7** *Reconstitution and Repackaging Computational Models*
- ✓ **Lecture 8** *Special Topics in Heat Transfer*

# Course Media

The CTA course will feature a multimedia delivery by mixing overheads, video animations and *InterLab sessions* (see next panel). Overheads will present integrated text, graphics and equations. The course emphasizes a strong pictorial representation, carefully defined nomenclature and transient animations. The entire presentation will be delivered electronically using screen projection of a laptop monitor.

**EVOLUTION**  
The Enhancement of Model Fidelity

- Evolve model fidelity with most important influence
  - not clear which aspect is most important
  - should perform *sensitivity studies* to estimate effects
  - suggest that convection on the top surface is most important
- Revise model with upper convection boundary and rerun simulation

**BENCHMARK SOLUTIONS**  
Semi-infinite Slab - Error Function Solution

- Initial and boundary condition data result in an exact solution

$$T(x, 0) = 0$$
$$T(0, t) = T_s \rightarrow T(x, t) = T_s \left( 1 - \operatorname{erf} \left( \frac{x}{\sqrt{4\alpha t}} \right) \right)$$
$$T(\infty, t) = 0$$

**TRANSIENT SOLUTIONS**  
Coldplate Example

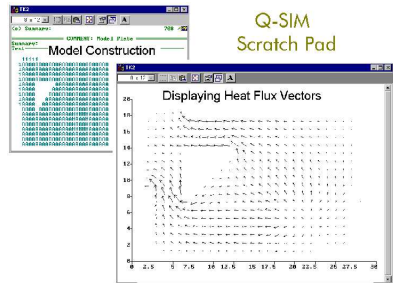
- Apply the coldplate discretization example to evaluate the solution progress
  - select a CV to observe the transient solution ~ CV-15 for example
  - progress through increasing values of time step

**Transient Animations**

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# InterLab with Q-SIM

The CTA course provides a unique integration of a laboratory environment with the compulsory question and answer sessions during lectures. During *InterLab*, which stands for *interactive laboratory*, the **Q-SIM** thermal analysis code will be applied to enhance the learning process by setting up fast computational examples. The simple scratch-pad environment of Q-SIM, allows a thermal problem to literally be built and analyzed in seconds. The screen projection will allow the students to observe model construction, solution and results analysis.



# Q-SIM Thermal Analyzer

Q-SIM which stands for *Quick Simulator* was designed and coded by the instructor. Built around an integrated model building, solving and post-processing environment, Q-SIM was used as the basis for developing content of the lecture series. Q-SIM provides a unique opportunity for the student to observe the coupling between the CTA fundamentals and code development and application.

# Course Length and Deliverables

Course length 2 days, 600 overheads  
Each student will receive the following:

- ✓ Complete BW hard copy set of course notes
- ✓ CD-ROM with overheads viewable with Adobe Acrobat® Reader™ (freely available from [www.adobe.com](http://www.adobe.com))
- ✓ Thermal animations (.AVI format) imbedded in overheads (keyed within Adobe Acrobat)

# CTA Course Syllabus

The CTA course is designed around 8 lectures which descend from a structured thermal modelling paradigm. The agenda of each lecture is presented in the following pages. By design, each lecture transitions from one into the next in roughly the same fashion that a systematic thermal analysis is performed.

Each of these lectures is then expanded into a series of detailed subjects. For example, Lecture 4 on discretization covers: *representation, motivation for discretization, connectivity, spatial fields, mathematical formulations, strategic discretization, refinement, heat flux laws, conductance models, enthalpy flux models, vectorization, sparse and flattened matrix representation*. This particular lecture contains approximately 90 overheads.

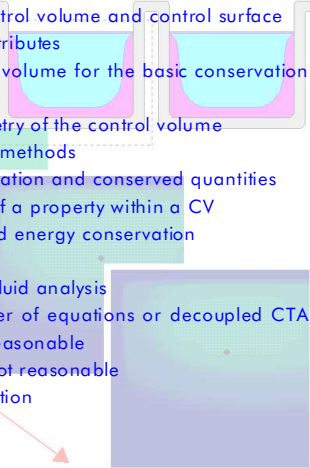
The CTA course will introduce the topics using practical thermal problems. For example, in lecture 4 a 2D coldplate is discretized using both the sparse and flattened matrix methods. While simple, this example is fruitful in revealing all of the important concepts. For example, both the sparse and flattened representations are setup in the classical explicit and implicit differencing methods in Lecture 5. Both are solved and stability regions are tested using matrix analysis software. With a slight change in one conductance value, stalled iteration is revealed which motivates the remaining discussion on semi-direct solution methods. Similarly, an analysis of numerical consistency is performed in Lecture 6 and finally model replication and reconstitution are demonstrated in Lecture 7.

## Lecture 1 Introduction to Computational Thermal Analysis (CTA)

- Introduction: Basic features of thermal analysis
- Structuring the analysis approach
- Model classification - positives and negatives
  - ↳ Experimental
  - ↳ Analytical
  - ↳ Computational
  - ↳ Hybrid Variants
- Emergence of CTA as a general purpose tool
- CTA as a community
- Structuring the analysis approach
  - ↳ Rationale
  - ↳ Difficulties and failings
- Example application of paradigm
  - ↳ Application problem: *formation of an ice cube*
  - ↳ composing the seven facets of the paradigm
  - ↳ Discussion
- CTA course syllabus for remaining lectures

## Lecture 2 Formulation of the Basic Equations of Heat Transfer

- An introduction to the control volume and control surface
  - ↳ the control volume attributes
  - ↳ surveying this control volume for the basic conservation process
  - ↳ simplifying the geometry of the control volume
  - ↳ relation to other CTA methods
- General conservation equation and conserved quantities
  - ↳ time rate of change of a property within a CV
  - ↳ mass, momentum and energy conservation
  - ↳ 3 equations to match
- Decoupling thermal and fluid analysis
  - ↳ simplifying the number of equations or decoupled CTA
  - ↳ when decoupling is reasonable
  - ↳ when decoupling is not reasonable
- Energy conservation equation
- Flux terms
  - ↳ the area vector
  - ↳ example of integrating flux quantities
- Volume source terms
- Final simplified equation
- Example application of energy equation
  - ↳ basic ODE
  - ↳ target solution state
  - ↳ concept of 1CV and 1 solution state
  - ↳ revealing important thermal features through discretization

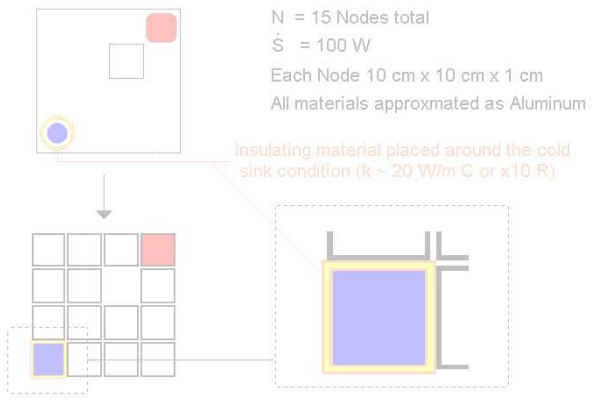


## Lecture 3 Decoupling Systems and Deriving Boundary Conditions

- Extent of full systems and the concept of a boundary condition
- Rendering boundary conditions by parting the full system into regions
- Types of boundary conditions
- Application study - Temperature boundary condition
- Application study - Heat flux boundary condition
- Controllability of temperature in heat flux boundaries
- Applicability of the Robin boundary condition
- Converting Robin boundary conditions

## Lecture 4 Discretization of the Governing Equations and Geometry

- Motivation for discretization
- Connectivity of control volumes and spatial fields
- Example: physical-spatial discretization and mathematical discretization
- Strategic discretization and refinement
- Cartesian based discretization to be applied
- Fitness evaluation - Cartesian grids to represent cylindrical systems
- Flux laws and control volume connectivity
  - ↳ thermal resistance
  - ↳ connecting corners
  - ↳ enthalpy flux laws
  - ↳ equivalence of upwind differencing of advection and one-way conductors
- Vectorization
  - ↳ control volume numbering
  - ↳ vectorized conservation equation
  - ↳ sparse and flattened vectorization
  - ↳ vectorized flux terms
  - ↳ conductance matrix
- Discretization example



## Lecture 5 Computational Solutions to the Discrete Equations

- Solving the conservation equations in unison
- Direct solutions to the vectorized conservation equation
  - ↳ the exponential matrix function
  - ↳ simplified steady state solutions
- Example: direct solution of vectorized coldplate example
  - ↳ survey of Eigenvalues for steady state
  - ↳ direct single inversion of conductance matrix
- Failure of direct methods
  - ↳ sparseness and memory limitations
  - ↳ search for new methods
- Introduction to time discretization
- Steady state solutions
  - ↳ evaluation for singular conductance matrix
  - ↳ singularity and the Neumann condition
  - ↳ residual forms
  - ↳ Generalized Newton's method
  - ↳ effect of relaxation factor
  - ↳ stopping criteria
- Transient solution methods - explicit differencing
  - ↳ explicit time differencing and the recursive vectorized conservation equation
  - ↳ transient solution instability as a function of time step
  - ↳ introduction to solution stability
- Stability analysis
  - ↳ modified conservation equation
  - ↳ stability assessment from Eigenvalues
  - ↳ error propagation equation
  - ↳ survey of Eigenvalues for coldplate example
  - ↳ r-factor assessment
  - ↳ controlling solution stability
- Transient solution methods - implicit differencing
  - ↳ Euler forward differencing
  - ↳ vectorized conservation equation
  - ↳ solution of the implicit temperature vector
  - ↳ hybrid direct inversion
- Example: coldplate simulation
  - ↳ transient solution at large  $\Delta t$
  - ↳ stability and Eigenvalue assessment
  - ↳ time step assessment
  - ↳ time accuracy and discussion
- Development of general time differencing schemes
  - ↳ Crank-Nicholson differencing for second order accuracy
  - ↳ accuracy and effect caused by splitting physics into explicit and implicit parts
  - ↳ emphasis on inverting systems  $Ax = f$
  - ↳ turning to numerical solution methods
- Basic relaxation methods
  - ↳ application to transient solutions
  - ↳ solution to coldplate example
  - ↳ retardation of solution rate due to conductance matrix
  - ↳ search for semi-direct inversion methods
- Survey and application of semi-direct inversion methods
- ADI methods
  - ↳ ADI Splitting

- Y Work units compared to SOR
- Y Banding effect caused by Splitting
- Y Implementation of orthogonal heat flow components
- Y ADI Brian Method
- Y Elimination of banding effect
- Y ADI as a steady state solver
- Conjugate gradient method
  - Y Method development
  - Y Pseudo code and discussion of pre-conditioners
  - Y Application to coldplate problem and other examples

## Lecture 6 Validation of Computational Models and Solutions

- Validation phase
  - Y a philosophy
  - Y types of errors
  - Y responsibility of the analyst
- Numerical consistency analysis
  - Y modified equation showing effect of finite CV size
  - Y effect of reducing CV size
  - Y mesh discretization factor
  - Y setup of the method
  - Y Example application of numerical consistency analysis
  - Y determining solution order from results
- Bench marking: exact solutions to benchmark transient solutions
- Showcase of various exact solutions with comparisons to numerical schemes
  - Y 1D slab, sine distribution
  - Y 1D slab semi infinite in length, error function solution
  - Y Burgers equation, steady and transient solutions and wave equations
  - Y phase change examples
- Energy survey methods for validation
- Qualitative energy surveys
  - Y approach
  - Y emphasis on continual model assessment, checking
  - Y applying heat flux vectors for rapid order-level model checks

- Quantitative energy surveys
  - Y defining subdomains
  - Y defining area vectors and flux quantities
  - Y example integration of heat flux over a subdomain
  - Y equivalent form using divergence theorem

shaded regions are averaged to determine the strategic point of comparison

Cold region boundary temperature replicated to same discretization factor. Only outside boundary CVs effect results as all T-BC are at 20 deg C (see lack of heat flow above)

## Lecture 7 Reconstitution and Repackaging Computational Models

- Model evolution and reconstitution
- Recognizing the model as an object
- Unplugging model from test boundary conditions
- Model replication and reissue of boundary conditions
- Example replication of the coldplate model with a coupled flow loop

## Lecture 8 Special Topics in Heat Transfer

- Modelling the advection terms from fluid flow
  - Y simple upwind and central differencing methods
  - Y implementation of advection term in SOR solutions
  - Y artificial numerical diffusion and dispersion errors
  - Y 3<sup>rd</sup> order upwind schemes
  - Y solution examples
- Erroneous CFD and CTA coupling via of divergent flows
- Flux integration
  - Y 1<sup>st</sup> and 2<sup>nd</sup> order conductance models
  - Y origin of negative conductors
  - Y structured and unstructured discretization
- Radiation heat transfer
- Phase change heat transfer
  - Y enthalpy methods
  - Y validation and solution examples
- Hyperbolic heat conduction
  - Y constitutive models
  - Y conservation laws
  - Y final equation
  - Y modelling techniques

